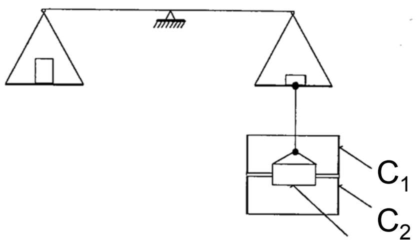
Athens Programme
Course CTU 1 - Metrology of Electrical Quantities

Topic 3

Methods for precision measurement of dc current and dc voltage. Modern potentiometers. Measurement of voltage, power and energy in audio-frequency range.

Current balance



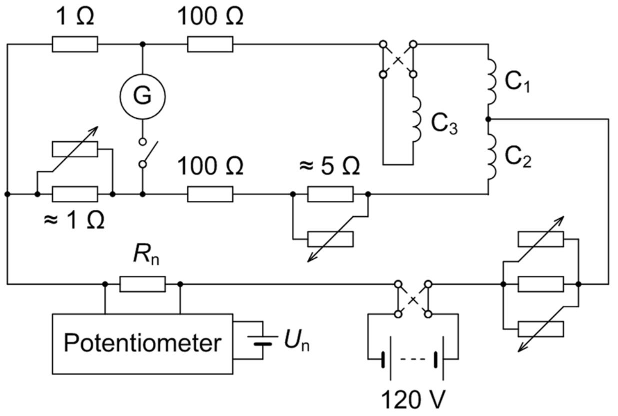
$I_{C1} = I_{C2} = I_{C3} = I$

I_{C1} enters C_1 at the bottom and leaves it at the top,

I_{C2} enters C_2 at the top and leaves it at the bottom. ⇒

⇒ Electromagnetic forces on the suspended coil, produced by these two currents, are of the same sense.

Current balance



Current balance

$$F_{13} = I_{C1}\, I_{C3}\, \frac{\partial M_{13}}{\partial z} = I^2 f_{13}$$

$$M_{13} = \frac{\mu\, \mu_0}{4\pi} \int_{C_1} \int_{C_3} \frac{d\sigma_1\, d\sigma_3}{a_{13}}$$

(Neumann´s formula)

Current balance

$$F_{23} = I_{C2}\, I_{C3} \frac{\partial M_{23}}{\partial z} = I^2 f_{23}$$

From symmetry,

$$f_{13} = f_{23} = f$$

Resulting electromagnetic force on C_3 is

$$F = F_{13} + F_{23} = 2 f I^2$$

Current balance operation

- The mass of the suspended coil plus the vertical electromagnetic force exerted on it are counterbalanced by means of a mass m on the left scale pan.
- After reversing the current in the suspended coil, the change Δm of this mass, necessary to offset the change ΔF of the acting electromagnetic force, is determined.

$$\Delta F = 4 f I^2 = \Delta m \cdot g \;\Rightarrow\; I = \frac{1}{2}\sqrt{\frac{\Delta m \cdot g}{f}}$$

Current balance operation

- A separate moving experiment makes it possible to avoid the troublesome calculation of f from the dimensions of the coils:

Coil C_3, which is threaded by the magnetic flux produced by the current I flowing in C_1 and C_2, is moved with a constant velocity v in the vertical direction, a voltage $u(t)$ being induced in it:

$$u(t) = 2I\frac{\partial M_{13}}{\partial t} = 2I\frac{\partial M_{13}}{\partial z}\frac{\partial z}{\partial t} = 2Iv\frac{\partial M_{13}}{\partial z}$$

Current balance operation

- If a voltage drop $U = R\,I$ is produced by the current I on a known resistance R and if a velocity v is chosen for which

$$\boxed{u(t) = U}$$

- in the moment of passage of the scale beam through its equilibrium position,

$$\boxed{f = \frac{\partial M_{13}}{\partial z} = \frac{R}{2v}}$$

Electronic kilogram

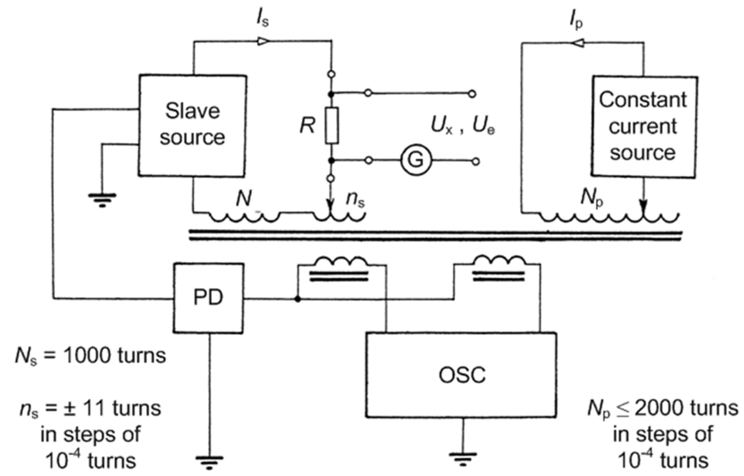
- In case that quantum standards of voltage and resistance are used to measure the current I flowing in the coils in course of the weighing experiment, a current balance with a known value of f can be used to produce known values of electromagnetic force and to monitor variations of masses which are used to counterbalance it.

Voltage balance

- is an apparatus based on counterbalancing the attractive force between electrodes of a capacitor.

$$F_z = \frac{1}{2} U^2 \frac{\partial C}{\partial z}$$

DC comparator potentiometer

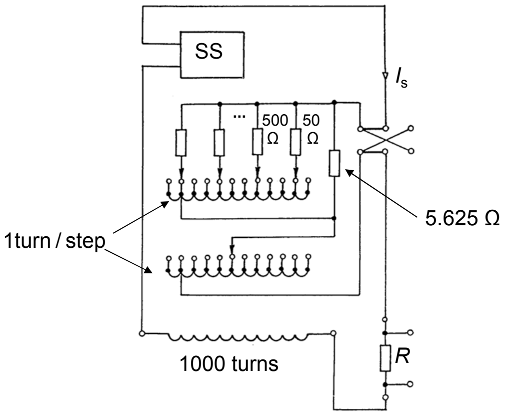


N_s = 1000 turns

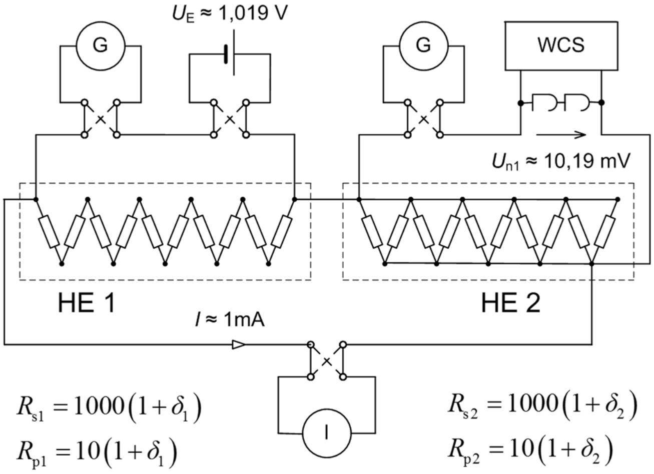
n_s = ± 11 turns in steps of 10^{-4} turns

$N_p \leq$ 2000 turns in steps of 10^{-4} turns

Realization of the equivalent of fractional turns



Josephson potentiometer



$$R_{s1} = 1000(1+\delta_1)$$
$$R_{p1} = 10(1+\delta_1)$$

$$R_{s2} = 1000(1+\delta_2)$$
$$R_{p2} = 10(1+\delta_2)$$

With Hamon standards interchanged

$$\boxed{\frac{U_\mathrm{E}}{R_{s2}} = \frac{U_\mathrm{n2}}{R_\mathrm{p1}}} \qquad \boxed{\frac{U_E}{R_{s1}} = \frac{U_{n1}}{R_{p2}}}$$

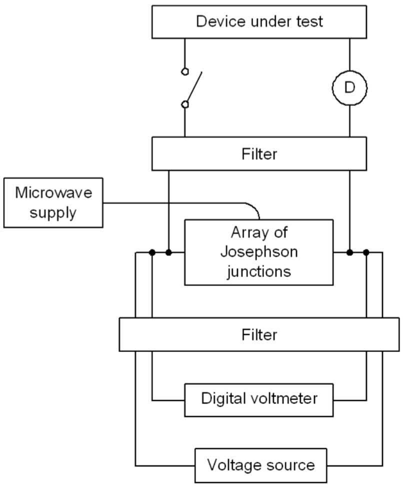
$$U_\mathrm{n1} + U_\mathrm{n2} = U_\mathrm{E}\left(\frac{R_\mathrm{p2}}{R_{s1}} + \frac{R_\mathrm{p1}}{R_{s2}}\right) = 0{,}01U_\mathrm{E}\left(\frac{1+\delta_2}{1+\delta_1} + \frac{1+\delta_1}{1+\delta_2}\right) =$$

$$= 0{,}01U_\mathrm{E}\frac{(1+\delta_2)^2 + (1+\delta_1)^2}{(1+\delta_1)(1+\delta_2)} = 0{,}01U_\mathrm{E}\frac{1+2\delta_2 \ +\delta_2^2 + 1 + 2\delta_1 \ +\delta_1^2}{1+\delta_1 + \delta_2 + \delta_1\delta_2} \doteq$$

$$\doteq 0{,}01U_\mathrm{E}\frac{2+2\delta_1 \ +2\delta_2}{1+\delta_1 + \delta_2} = 0{,}02U_\mathrm{E}$$

$$\boxed{U_\mathrm{E} = 50(U_\mathrm{n1} + U_\mathrm{n2})}$$

Calibration of an electronic voltage standard



Device under test

D

Filter

Microwave supply

Array of Josephson junctions

Filter

Digital voltmeter

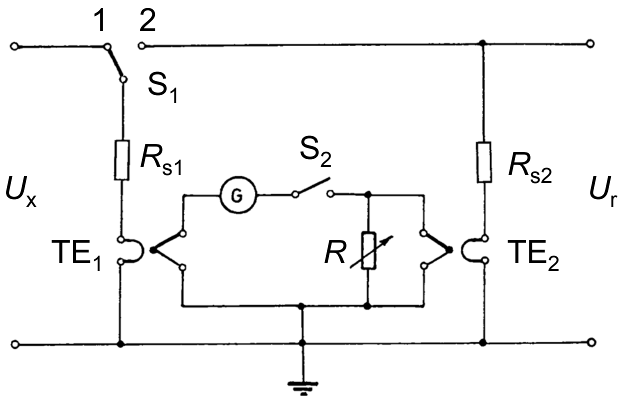
Voltage source

Primary voltage standard system with closed-cycle refrigerator

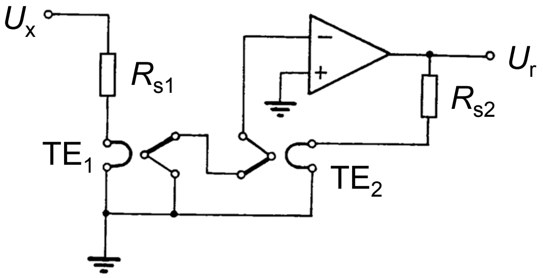
Primary voltage standard system with closed-cycle refrigerator

- 100 nV calibration accuracy
- -11 V to 11 V range available from 10-V system
- typically 1 hour stability time at 10 V for 10-V system
- automatic voltage calibration in minutes
- automatic calibration of DVMs
- complete system diagnostics

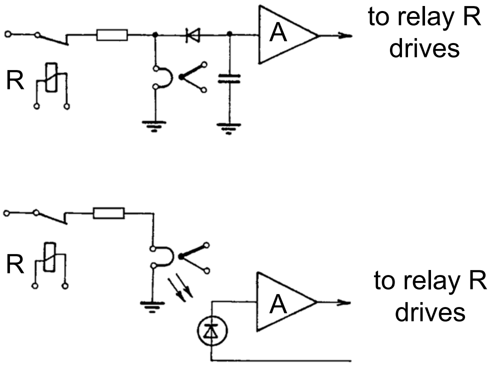
RMS/DC thermocouple comparator



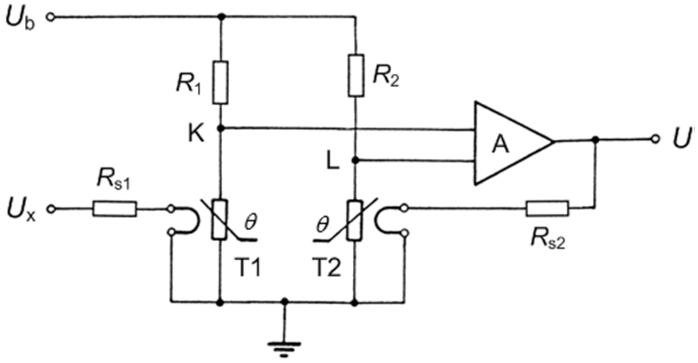
Automatic RMS/DC thermocouple comparator



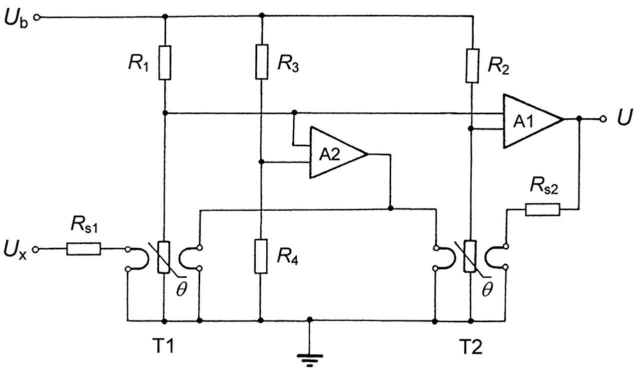
Thermocouple protection



RMS/DC thermistor comparator



Improved RMS/DC thermistor comparator



Definitions for powers

Sinusoidal voltages and currents

Active power : $P = U\, I \cos\varphi$

Apparent power : $S = U\, I$

Reactive power : $Q = U\, I \sin\varphi$

$$S^2 = P^2 + Q^2$$

Definitions for powers

Nonsinusoidal voltages and currents

$$U = \sqrt{\sum_{n=1}^{\infty} U_n^2} \quad \text{and} \quad I = \sqrt{\sum_{n=1}^{\infty} I_n^2}$$

Apparent power : $S = U\, I$

Active power : $P = \sum_{n=1}^{\infty} U_n\, I_n \cos\varphi_n$

Reactive power : a number of different definitions have been proposed

Definitions for powers

Budeanu´s definition for reactive power :

$$Q_{\mathrm{B}} = \sum_{n=1}^{\infty} U_n \, I_n \sin\varphi_n$$

$$S^2 = P^2 + Q_{\mathrm{B}}^2 + D_{\mathrm{B}}^2$$

Distortion power :

$$D_{\mathrm{B}} = \sqrt{S^2 - P^2 - Q_{\mathrm{B}}^2}$$

Practical definitions for powers proposed by the IEEE Working Group

on Nonsinusoidal Situations

$$U^2 = U_1^2 + U_{\mathrm{H}}^2 = U_1^2 + \sum_{n \neq 1} U_n^2$$

$$I^2 = I_1^2 + I_{\mathrm{H}}^2 = I_1^2 + \sum_{n \neq 1} I_n^2$$

$$(U\,I)^2 = (U_1\,I_1)^2 + (U_1\,I_{\mathrm{H}})^2 + (U_{\mathrm{H}}\,I_1)^2 + (U_{\mathrm{H}}\,I_{\mathrm{H}})^2$$

Practical definitions for powers proposed by the IEEE Working Group

Fundamental apparent power :

$$\left(U_1 I_1\right)^2 = S_1^2 = P_1^2 + Q_1^2 =$$

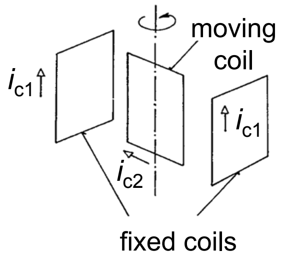
$$= \left(U_1 I_1 \cos\varphi_1\right)^2 + \left(U_1 I_1 \sin\varphi_1\right)^2$$

Non-fundamental apparent power :

$$S_\mathrm{N}^2 = (U_1 I_\mathrm{H})^2 + (U_\mathrm{H} I_1)^2 + (U_\mathrm{H} I_\mathrm{H})^2 = S^2 - S_1^2$$

Nonactive power : $N^2 = S^2 - P^2$

Electrodynamic method of measurement of AC power



$$\text{Inst. torque} = i_{c1} i_{c2} \frac{\mathrm{d}M}{\mathrm{d}\alpha}$$

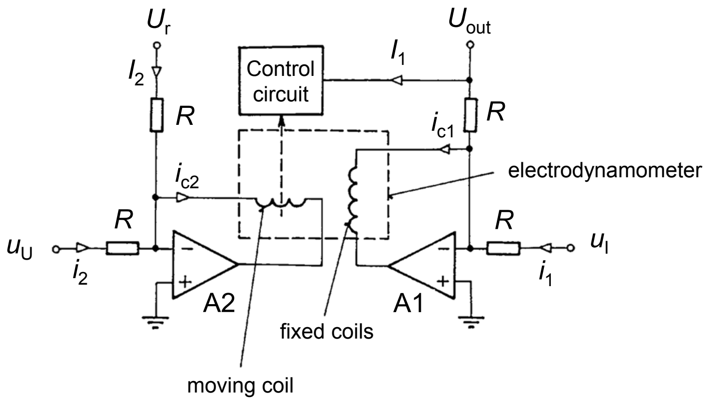
$$\text{mean torque} = \langle i_{c1} i_{c2} \rangle \frac{\mathrm{d}M}{\mathrm{d}\alpha}$$

$$\langle i_{c1} i_{c2} \rangle = \frac{1}{T} \int_0^T i_{c1} i_{c2}\, \mathrm{d}t$$

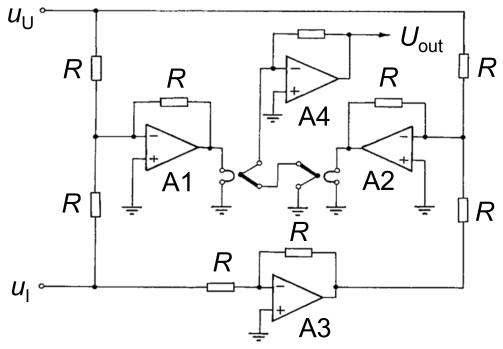
$$i_{c1} = i_1 + I_1$$
$$i_{c2} = i_2 - I_2$$

$$\langle i_1 i_2 \rangle = I_1 I_2 \;\Rightarrow\; \text{mean torque} = 0 \;\Rightarrow\; \alpha = 0$$

Electrodynamic method of measurement of AC power



Thermal wattmeter



$$U_{out} = k\left\langle (u_U + u_I)^2 - (u_U - u_I)^2 \right\rangle = 4\,k\,\langle u_U\, u_I \rangle$$

Digital sampling method

$$P = \frac{1}{T}\int_0^T u(t)\,i(t)\,\mathrm{d}t = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t$$

$$p(t) = P + \sum_{k=1}^{\infty} P_k \sin\left(2\pi k \frac{t}{T} + \varphi_k\right)$$

$$\hat{P} = \frac{1}{n}\sum_{l=0}^{n-1} u(t_l)\; i(t_l) = \frac{1}{n}\sum_{l=0}^{n-1} p(t_l)$$

$$\boxed{\left|\Delta P\right| = \left|\hat{P} - P\right| \le \sum_{k>0}^{*} P_k}$$

The starred summation is over only those power harmonics whose frequencies are integer multiples of the sampling frequency.