Athens Programme
Course CTU 1 - Metrology of Electrical Quantities

Topic 2

Voltage and current inductive ratio devices and optimization of their metrological parameters

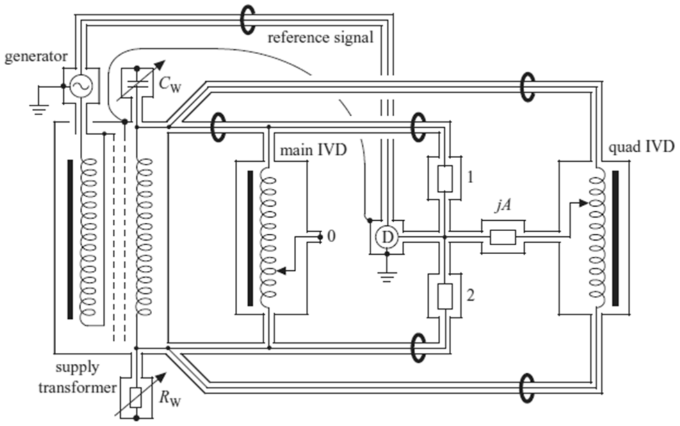
---

Content

- **Inductive voltage divider (IVD)**
    - Principle
    - Errors of IVD's
    - Two-stage IVD (magnetizing winding)
- **AC current comparator**
    - Principle, properties, applications
- **DC current comparator**
    - Principle, properties, applications
- **Cryogenic current comparator (CCC)**
    - Principle of CCC
    - SQUID – basic function, SQUID readout electronics
    - Applications – DC resistance ratio measurement bridge

Why we need inductive voltage dividers …

- **A two-terminal-pair bridge based on IVDs** – circuit for very precise impedance measurement at level of a few ppm or better !

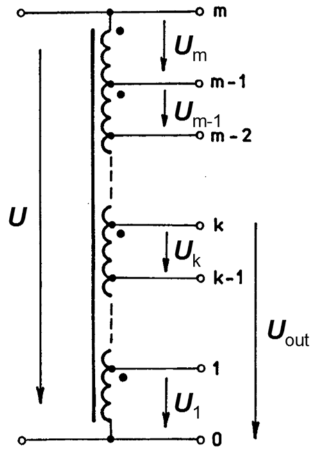


Simple inductive voltage divider (IVD)

- Key-component of all transformer bridges for impedance comparison
- Used for division of input voltage *U*
- Looks like a autotransformer
- To provide fine resolution of its voltage ratio more decades must be used

$$\boldsymbol{U}_{\text{out}} = \sum_{i=1}^{k} \boldsymbol{U}_{\text{i}} = D\left(1 + \alpha_{\text{D}} + \text{j}\beta_{\text{D}}\right)\boldsymbol{U}$$

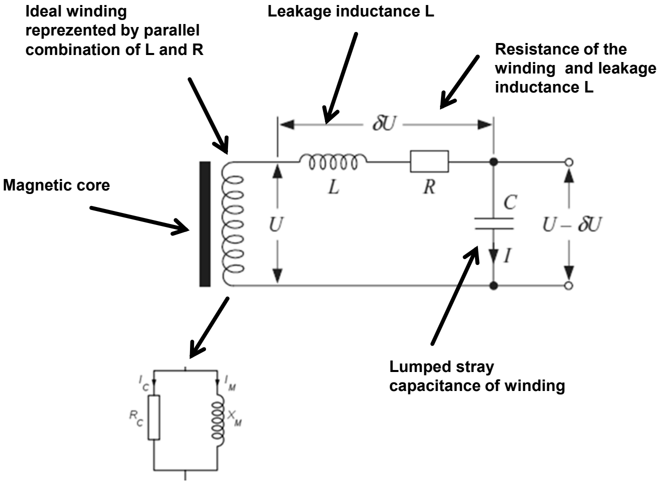
$$D = k/m$$



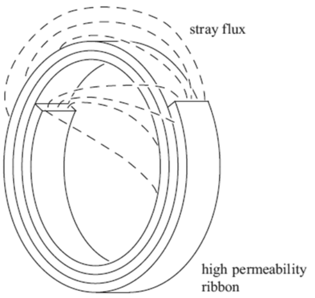
Simple one-stage IVD with m sections

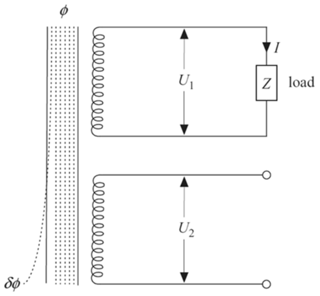
Electrical model of one section of a one-stage IVD

The effect of leakage inductance, capacitance and resistance



Origin of leakage inductance

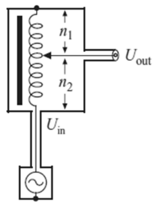




$$-d(\delta\phi)/dt = L\frac{dI}{dt}$$

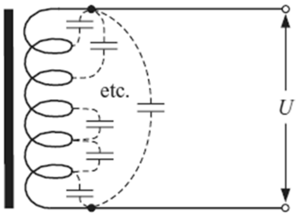
⇩

$$small\ stray\ flux\ affects\ voltage\ ratio\ \frac{n_1}{n_2}$$



Origin of parasitic capacitances

- Each turn has a capacitance to the IVD terminals
- Each winding section has a capacitance to another section
- All distributed capacitances can be represented by a single lumped capacitance C
- Capacitance C forms with main magnetising inductance **parallel resonant circuit** – it should be as small as possible to obtain high resonance frequency f_{rez} $f_{ref} = \frac{1}{2\pi\sqrt{LC}}$
- Capacitance may be reduced by electrostatic shielding

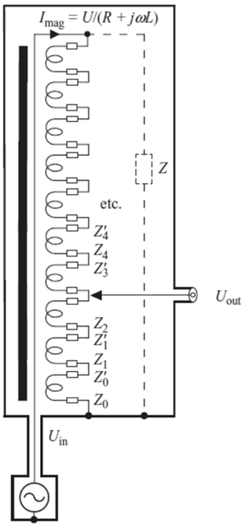


Erors of IVD - main causes

- **In low-frequency range**
    - Errors are mainly due to unequal voltage drops which are produced by the magnetizing current when it passes through unequal resistances and leakage inductances of divider sections.
- **In high-frequency range (over 10 kHz)**
    - Errors are caused mainly by various parasitic capacitances between sections as well as between the turns.
- **Errors coming from IVD output loading**
    - According to the IVD model – its output has a finite output impedance -> there is always a need to use IVD in a such way that output is not loaded by connected measurement circuits!

One-stage divider – problem of input impedance

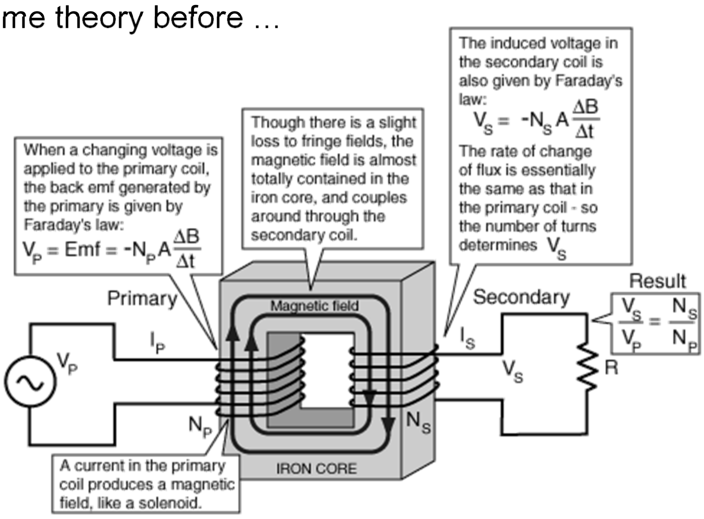
Equivalent circuit of one-stage divider



- Input impedance of a single-stage IVD is given by magnetising impedances of all sections and all series impedances Z_i, Z_i' (leakage inductances and resistances of the windings)
- $Z_{in} = U_{in} / I_{mag}$
- For applications in transformer bridges, we need IVDs with very high input impedances.
- **How do we obtain them ? By use of TWO-STAGE dividers.**

Two-stage divider

- Some theory before …



When a changing voltage is applied to the primary coil, the back emf generated by the primary is given by Faraday's law: $V_P = \text{Emf} = -N_P A \frac{\Delta B}{\Delta t}$

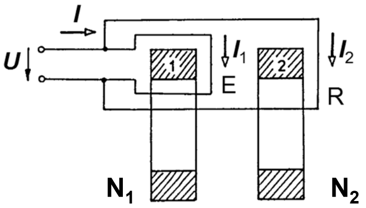
Though there is a slight loss to fringe fields, the magnetic field is almost totally contained in the iron core, and couples around through the secondary coil.

The induced voltage in the secondary coil is also given by Faraday's law: $V_S = -N_S A \frac{\Delta B}{\Delta t}$ The rate of change of flux is essentially the same as that in the primary coil - so the number of turns determines V_S

A current in the primary coil produces a magnetic field, like a solenoid.

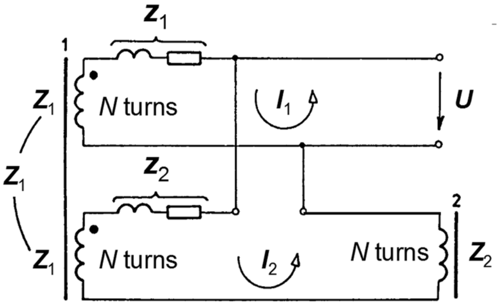
Two-stage divider

- In comparison with one-stage divider - **one magnetic core and one winding** must be added
- Two-stage IVD is equipped by an exciting winding (E) and a ratio winding (R)
- EW being wound around core no. 1 only, RW is wound around both cores
- Number of turns of ER = number of turns of RW ($N_1 = N_2$)!



$$\Phi_1 = \Phi_1 + \Phi_2 \Rightarrow \quad \Phi_2 = 0 \Rightarrow I_2 = 0$$

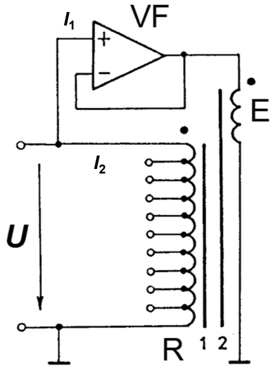
Equivalent circuit of the two-stage divider



$$I \cong \frac{U}{Z_1}\ , \quad \frac{I_2}{I_1} \cong \frac{z_1}{Z_2}$$

Improved two-stage divider

- Voltage follower is used for driving EW
- Due to its high input impedance (>10GΩ) also the input impedance of the whole circuit is very high



Calibration of IVDs

- **Example 1** – IVD having 1 decade and 11 output taps – voltage of each section is 1/11 of the input voltage – number of calibration points is 11

- **Example 2** – IVD having 8 decades and 1 output tap - number of all possible settings is so high that complete calibration is practically impossible
  - Solution: perform calibration only for:

$i/11$, $i = 1, 2, ..., 10$

i.e.

1/11 = 0.090 909 09
2/11 = 0.181 818 18
3/11 = 0.272 727 27
4/11 = 0.363 636 36
5/11 = 0.454 545 45

etc.

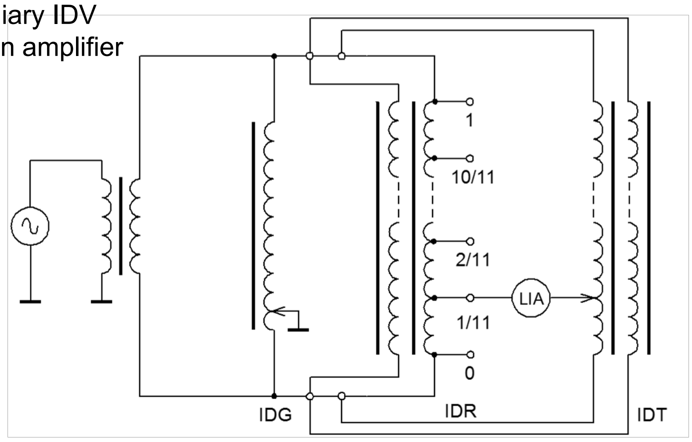
  - Reason: during calibration process each setting of each decade is verified

Calibration procedure

- **Step 1** - comparison of the divider under test with an 11 section reference divider
- **Step 2** - calibration of the reference divider based on employment of an auxiliary 11:1 transformer (it is not necessary to know the exact value of the transformer ratio before the experiment)

Comparison of two-stage IVDs

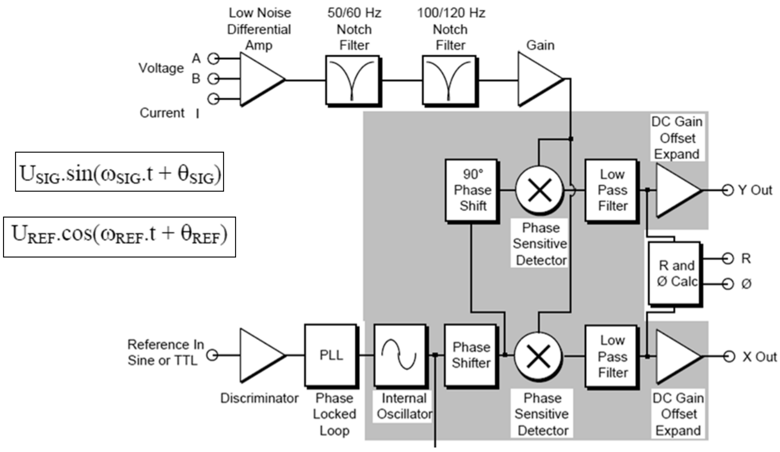
- IDT – IVD under TEST
- IDR – reference IVD(divider with known errors)
- IDG – auxiliary IDV
- LIA – lock-in amplifier



- For clarity all cable shieldings are omitted

Principle of the lock-in amplifier (LIA)

- LIA – a very often used scientific instrument
- typically used as a null detector in impedance metrology
- it allows to detect signals 100x times smaller than noise (40 dB)



$U_{SIG}.\sin(\omega_{SIG}.t + \theta_{SIG})$

$U_{REF}.\cos(\omega_{REF}.t + \theta_{REF})$

What is on the LIA outputs ?

$$U_X = \tfrac{1}{2} U_{SIG}\, U_{REF} \cos [\,(\omega_{SIG} - \omega_{REF}).t + \theta_{SIG} - \theta_{REF}] - \tfrac{1}{2} U_{SIG}\, U_{REF} \cos [\,(\omega_{SIG} + \omega_{REF}).t + \theta_{SIG} + \theta_{REF}]$$

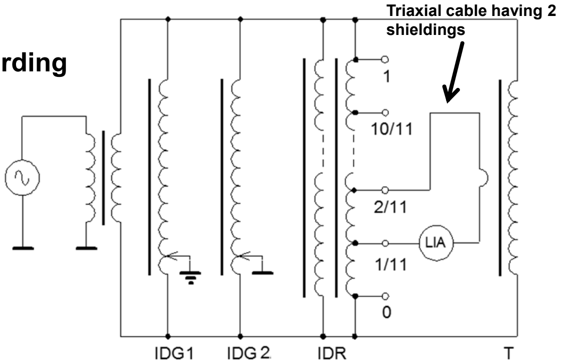
$$U_Y = \tfrac{1}{2} U_{SIG}\, U_{REF} \sin [\,(\omega_{SIG} + \omega_{REF}).t + \theta_{SIG} + \theta_{REF}] + \tfrac{1}{2} U_{SIG}\, U_{REF} \sin [\,(\omega_{SIG} - \omega_{REF}).t + \theta_{SIG} - \theta_{REF}]$$

$$U_X = \tfrac{1}{2} U_{SIG}\, U_{REF} \cos [\theta_{SIG} - \theta_{REF}]$$
$$U_Y = \tfrac{1}{2} U_{SIG}\, U_{REF} \sin [\theta_{SIG} - \theta_{REF}]$$

Calibration of the reference divider

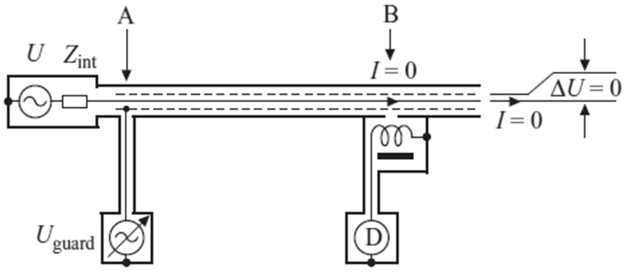
- T – auxiliary transformer with nominal ratio of 1/11
- IDR – reference IVD for calibration
- IDG1 – auxiliary IVD – its output defines potential of the shield of the whole circuit
- IDG1 – auxiliary IVD – its output defines potential of the inner shielding of the triaxial cable
- LIA – lock-in amplifier

- **Principle** of **active guarding** is applied to avoid loading by parasitic cable capacitances


Triaxial cable having 2 shieldings

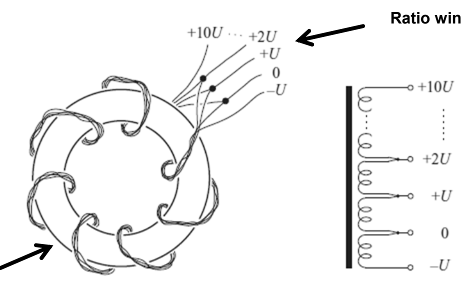
Principle of active guarding (AG)

- Coaxial cables with a characteristic impedance of 50 ohm have a typical parasitic capacitance of 100pF/m
- For application of the AG, we need a triaxial cable and some auxiliary source U_{guard} which **maintains the potential of the intermediate shield** so that voltage between this shield and the inner wire is a zero (*$\Delta U = 0$ V*)
- Then no current flows through the parasitic capacitance and the source *U* is not loaded



How to create IVD with fixed output taps?

- Tape-wound toroidal core made from thin strip of a soft magnetic material with sufficiently high permeability (e.g. of Supermalloy, Mumetal or amorphous materials for HF).
- A rope of **m** strands wound a sufficient number around the core; the ends are soldered together to form a continuous ratio winding.

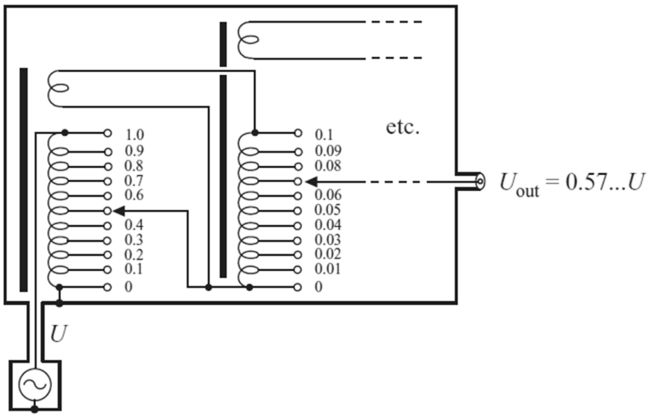


Ratio windings

Magnetic cores with high permeability

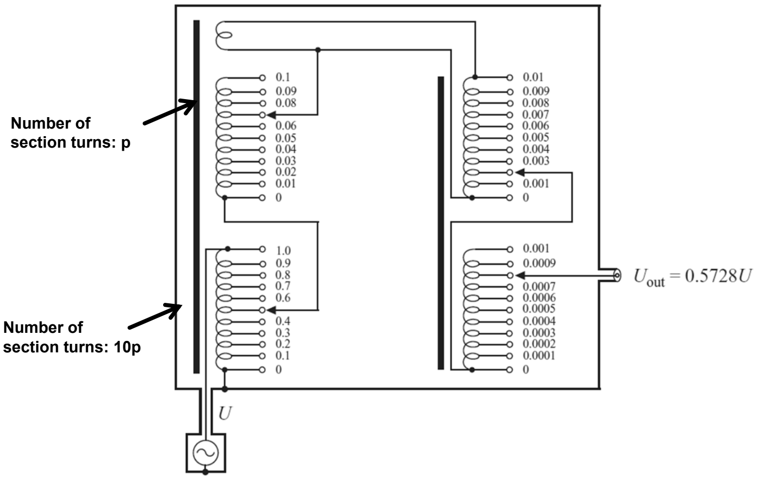
Multi-decade Kelvin-Varley divider – variant I

- When we need finer resolution of the divider ratio
- Variant I**: each section = one magnetic core**



Multi-decade Kelvin-Varley divider – variant II

- Variant II – economic **- each successive pair of decades wound onto one magnetic core**



$U_{out} = 0.5728U$

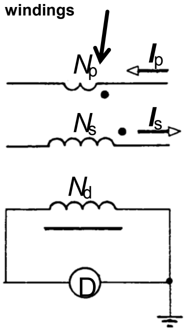
Number of section turns: p

Number of section turns: 10p

Simple AC current comparator

- device for comparison of **two AC currents**
- both currents create magnetic fluxes in the core in opposite direction
- at balance – magnetomotive force coming from **current Ip** is the same as magnetomotive force coming from **current Is**

Red point means beginning of windings



primary winding

secondary winding

detection winding

toroidal magnetic core

detector

$$N_p I_p - N_s I_s = 0$$

Errors of current comparators – main causes

- capacitance coupling between detection and ratio windings
  - can be eliminated by electrostatic shielding

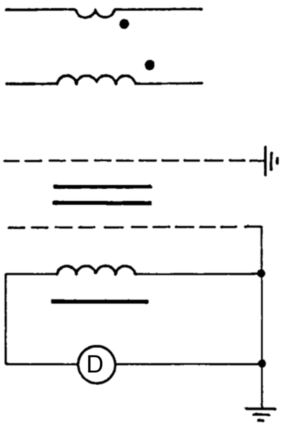
- stray magnetic flux coming from ratio windings
  - can be eliminated by application of a magnetic shield

- parasitic capacitances of the windings
  - their effect is more pronounced when the number of turns increases
  - when a large number of turns must be used to achieve high sensitivity, application of a compensation winding is advisable

---

AC current comparator with shields

- Windings and shields of a a AC comparator



primary winding

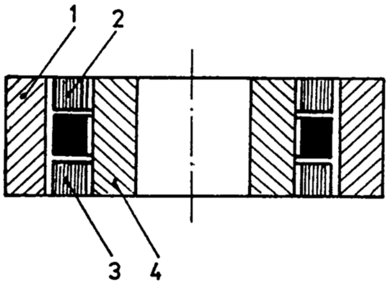
secondary winding

copper shield
magnetic shield
copper shield
detection winding
toroidal magnetic core

detector

Magnetic shield consisting of four toroids, 1 - 4

- All toroids are realized from ribbon having high initial permeability
- By means of this shield, leakage magnetic fluxes are kept from reaching the detector winding.

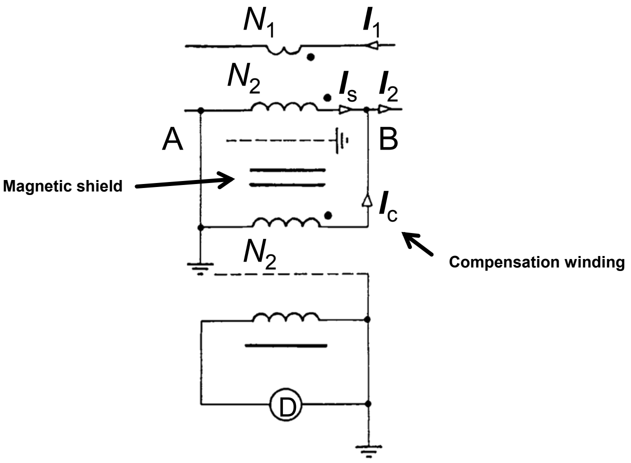


Error of an AC current comparator

- Where ***n*** is the turns ratio of the ratio windings and $\boldsymbol{I}_\mathrm{s}$, $\boldsymbol{I}_\mathrm{p}$ are the currents in ratio windings measured at their respective marked terminals when these terminals are at ground potential

$$\boldsymbol{I}_\mathrm{s} = \frac{\boldsymbol{I}_\mathrm{p}}{n}\left(1 + \boldsymbol{\varepsilon}\right)$$

Compensated current comparator



Magnetic shield

Compensation winding

Compensated current comparator

- At balance, there is no flux in the toroidal magnetic core, the resultant magnetizing m.m.f. for this core being

$$N_1 \boldsymbol{I}_1 - N_2 \boldsymbol{I}_\text{s} - N_2 \boldsymbol{I}_\text{c} = 0\ .$$

- The resultant magnetizing m.m.f. for the shield is

$$\boldsymbol{U}_\text{m.s.} = N_1 \boldsymbol{I}_1 - N_2 \boldsymbol{I}_\text{s} = N_2 \boldsymbol{I}_\text{c}$$

- the corresponding shield flux is

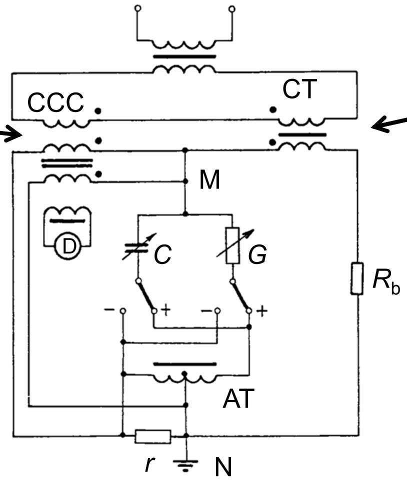
$$\boldsymbol{\Phi}_\text{m.s.} = \boldsymbol{U}_\text{m.s.} / R_\text{m.s.} = N_2 \boldsymbol{I}_\text{c} / R_\text{m.s.}$$

Compensated current comparator

- A voltage induced by $\boldsymbol{\Phi}_{\text{m.s.}}$ into the secondary winding nearly cancels all voltage drops between the points A and B, namely
  - the drop on resistance of the secondary winding,
  - the drop on leakage inductance of the secondary winding,
  - the drop on possible burden impedance in series with the secondary winding.

Application of AC current comparators

- Calibration of a current transformer



Compensated current comparator

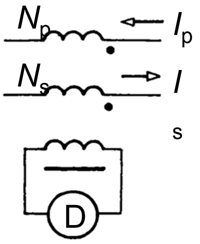
Current transformer under TEST

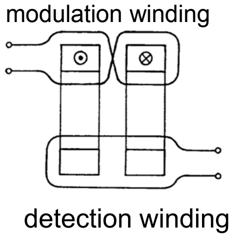
$$\varepsilon_{\mathrm{CT}} \cong \underbrace{(G + \mathrm{j}\omega C)\, r}_{\text{Ratio error of CT}}$$

u=

Principle of the DC current comparator

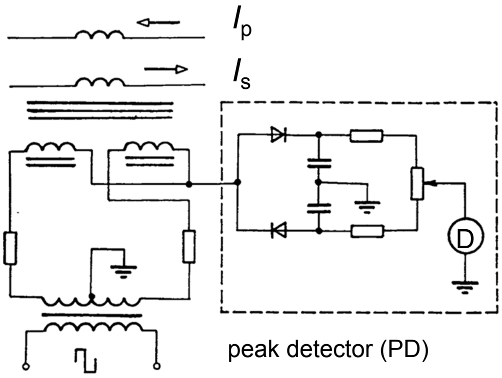
- Device for comparison of two DC currents
- If some DC current flows through a winding , the corresponding DC magnetic flux core is created in the core.
- However DC magnetic flux does not induce any voltage in the detection winding
- Modulation winding must be used !

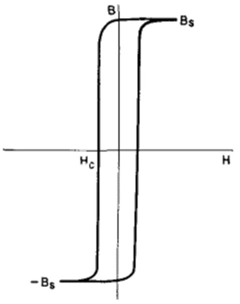




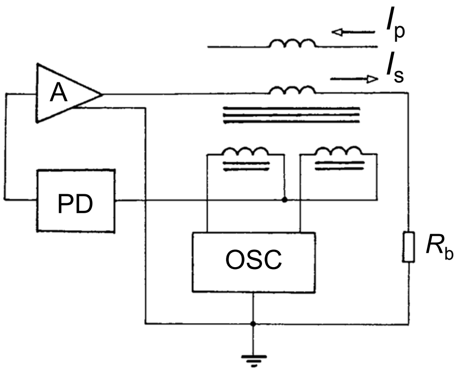
$$u=-N\frac{d(\phi)}{dt}=N\frac{d(BS)}{dt}=NS_{FE}\frac{d(\mu H)}{dt}=NS_{FE}[\mu\frac{\delta(H)}{\delta t}+H\frac{\delta(\mu)}{\delta t}]$$

DC current comparator





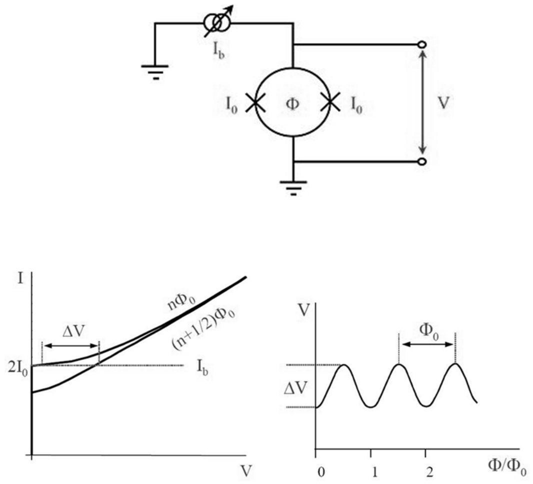
Automatically balanced DC current comparator



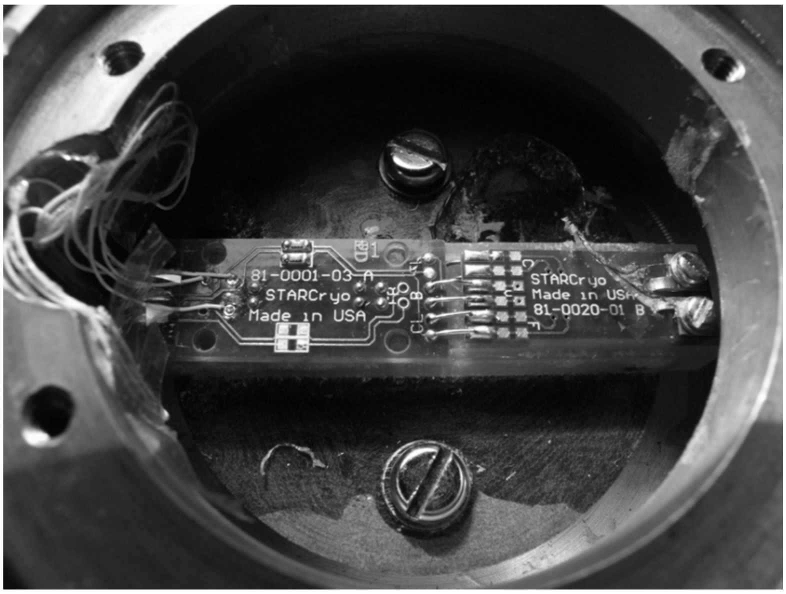
Cryogenic current comparator (CCC)

- Device used in precision electrical measurements to compare two electrical currents at the highest level of accuracy (10^{-9}) !!!
- Typical application of the CCC: **comparison of resistance standards**
- Two reasons why the CCCs are so sensitive:
  - all ratio windings are put into a superconducting shield ensuring perfect isolation from all external magnetic fluxes (based on Meissner effect)
  - for detecting unbalanced magnetic flux the most sensitive sensor of DC magnetic flux - DC SQUID -is used

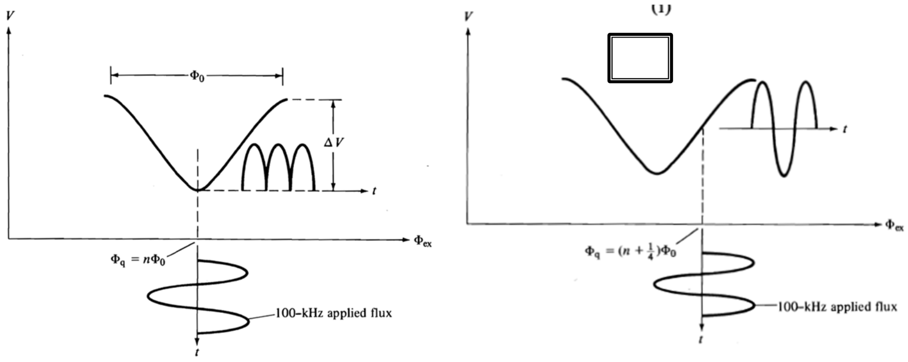
DC SQUID Principle



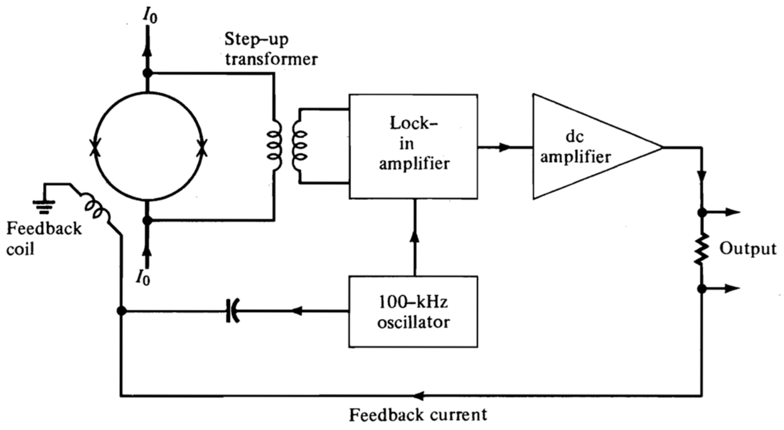
DC SQUID - Superconducting QUantum Interference Device



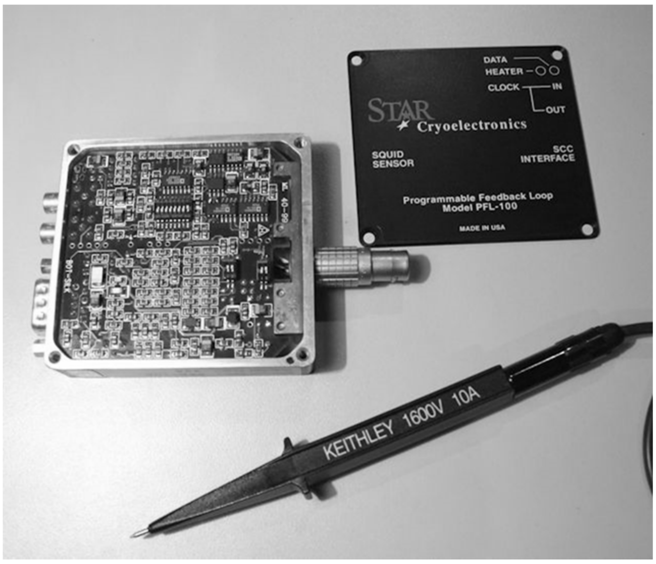
Detection of the signal from SQUID



Readout electronics for SQUID

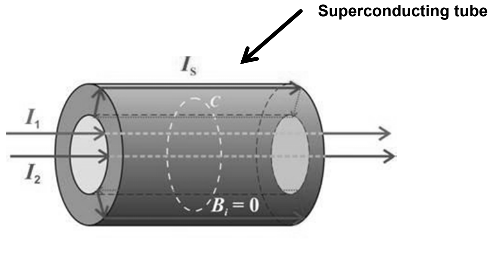


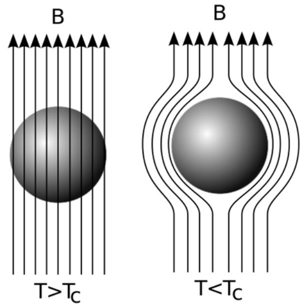
SQUID's readout electronics – commercial product



Superconducting shield - Meissner effect application

- Due to the Meissner effect, no magnetic flux can be inside the superconductor - the space inside the tube is perfectly shielded

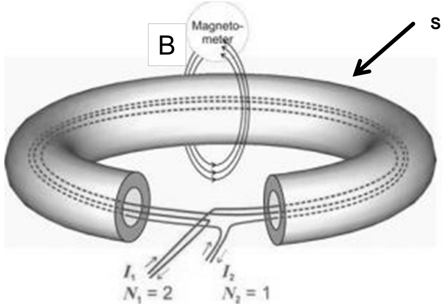




$$\int B \cdot ds = 0 = \mu_0 (I_1 + I_2 + I_s)$$

Principle of comparison two currents

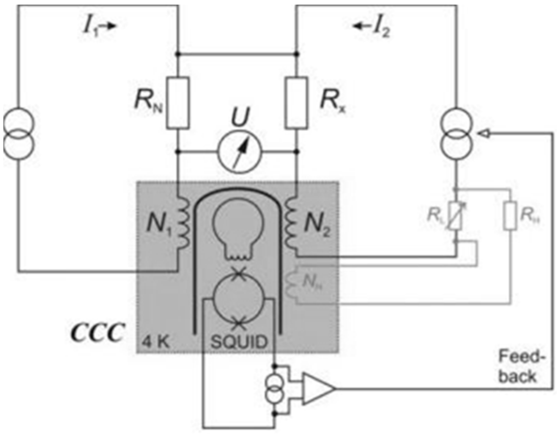
- The CCC is balanced if $N_1 I_1 - N_2 I_2 = 0$



Superconducting shield

- According to the Ampere's law $\int_C B \cdot ds = \mu_0 \sum_i I_i$
- Balance occurs when the magnetic flux density B = 0
- DC SQUID is used the detector of magnetic field

Application of CCC – DC resistance ratio bridge



- circuit for comparison of two resistance standards: R_X, R_N
- two isolated precise current sources
- as a detector of zero difference of voltage drops across the resistance standards, a low noise nV electronic module (A20) is used